Short Papers

Glove-Based Approach to Online Signature Verification

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Abstract—Utilizing the multiple degrees of freedom offered by the data glove for each finger and the hand, a novel online signature verification system using the Singular Value Decomposition (SVD) numerical tool for signature classification and verification is presented. The proposed technique is based on the Singular Value Decomposition in finding r singular vectors sensing the maximal energy of glove data matrix $\boldsymbol{A}$, called principal subspace, so the effective dimensionality of $\boldsymbol{A}$ can be reduced. Having modeled the data glove signature through its r-principal subspace, signature authentication is performed by finding the angles between the different subspaces. A demonstration of the data glove is presented as an effective high-bandwidth data entry device for signature verification. This SVD-based signature verification technique is tested and its performance is shown to be able to produce Equal Error Rate (EER) of less than 2.37 percent.

Index Terms—Data glove, online signature verification, singular value decomposition.

1 Introduction

SIGNATURE verification can be split into two categories: *Static or offline*: In this mode, users write their signature on paper, digitize it through an optical scanner or comer, and a biometric system recognizes the signature by analyzing its shape. *Dynamic or online*: In this mode, users write their signature on a digitizing tablet, smart pen, or pen tablet. Considering the highest security levels required by the online systems (dynamic), most of the efforts of the researchers in this field address this group.

The design of *a dynamic signature verification system* initially involves the following four aspects:

1. data acquisition and preprocessing (input device),
2. feature extraction,
3. matching (classification), and
4. decision making.

Fig. 1 shows the general online signature verification process.

Although successful in resisting attempts by imposters, the Dynamic Signature Verification (DSV) system still faces serious challenges for various reasons. The large variation in the execution speed of various phases of a signature is one such reason. Another reason is the quality and positions of the physical properties describing the signature themselves. Other factors affecting the difficulty of DSV are the emotional state of the signing person and the accuracy of the input device used. For successful implementation, any designed DSV system must take into consideration all these factors.

In this paper, a novel online signature verification system using data glove as an input device and the SVD numerical tool for modeling and matching is proposed.

This paper is organized as follows: Section 2 describes the details of the proposed SVD-based signature verification technique. Section 3 shows experimental results including the selection of the system parameters and a comparison with other online techniques. Section 4 concludes the paper.

For clarity, an attempt has been made to adhere to a standard notational convention. Lower case **boldface** characters will generally refer to vectors. Upper case **BOLDFACE** characters will generally refer to matrices. Vector or matrix transposition will be denoted using $(.)^{\mathrm{T}}$. $\mathbf{R}^{\mathrm{n}}$ denotes the real vector space of n dimensions.

2 The SVD-Based Data Glove Signature Verification System

Data glove [1] is a new dimension in the field of virtual reality environments, initially designed to satisfy the stringent requirements of modern motion capture and animation professionals. It offers comfort, ease of use, a small form factor, and multiple application drivers. The high data quality, low cross-correlation, and high data rate make it ideal for online signature verification systems. The dynamic features of the data glove provide information on:

1. Patterns distinctive to an individuals' signature and hand size.
2. Time elapsed during the signature process.
3. Hand trajectory dependent rolling.

Thus, while most input devices offer few degrees of freedom, the data glove is unique in offering multiple degrees of freedom in that it provides data on both the dynamics of the pen motion during the signature and the individual's hand shape. Fig. 2 shows the 5DT Data Glove 14 Ultra with the location of the sensors [1].

In the next section, we describe the proposed approach for the data glove input device. The approach is based on the singular value decomposition (SVD) in reducing the dimensionality of the data glove output matrix, and extracting a number of unique features (Modeling) for signature classification (matching).

2.1 The r-principal Subspace for Features Extraction

In signature verification and other signal processing applications, vector sequences are measured or computed. Such a situation arises whenever multivariable signals are measured in time and at fixed locations. For the analysis of such data sequences, a wide variety of multivariate analysis tools are available. The underlying theme of much multivariate analysis is simplification and explanation of the observed phenomena.

Consider a data glove with m sensors each of which generates n samples per signature, producing an output data matrix, $\boldsymbol{A}(m \times n)$. Usually $n \gg m$, where m denotes the number of measured channels while n denotes the number of measurements. It has been found in many signal processing applications and control systems that the singular value decomposition of matrix formed from observed data can be used to improve methods of signal parameter estimation and system identification. In this section, we extend the implementation of SVD and the principal components of data matrix $\boldsymbol{A}$ towards signature verification systems.

Principal component analysis, originating in work by Karl Pearson around the turn of the last century and further developed in 1930s by Harold Hoteling, consist of finding an orthogonal transformation of the original-stochastic-variables to a new set of uncorrelated variables, which are derived in nonincreasing order of importance. These so-called principal components are linear

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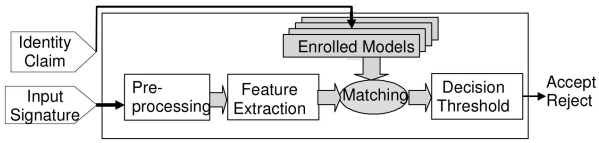

Fig. 1. Online signature verification system.

combination of the original variables such that its first few components will account for most of the variation in the original data so the effective dimensionality of the data can be reduced [2]. Since, the concept of oriented energy is closely related to principal components analysis, we start our work using this definition.

2.1.1 Oriented Energy

The column vectors of an $m \times n$ matrix $\boldsymbol{A}$ are considered to form an indexed set of m vectors, denoted by $\{\boldsymbol{a}_k\}$, $k = 1, 2, \ldots, n$. An m-dimensional vector $\boldsymbol{q}$ and the direction it represents in a vector space are used as synonyms.

Definition 1. *Energy of a vector sequence.*

Consider a sequence of m-vectors $\{\boldsymbol{a}_k\}$, $k = 1, 2, \ldots, n$ and associated real $m \times n$ matrix $\boldsymbol{A}$. Its total energy $E[\boldsymbol{A}]$ is defined via Frobenius norm of the $m \times n$ matrix $\boldsymbol{A}$

$$E[\boldsymbol{A}] = \|\boldsymbol{A}\|_F^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{a}_{ij}^2. \tag{1}$$

Definition 2. *Oriented energy.*

Let $\boldsymbol{A}$ be a $m \times n$ matrix and denote its n column vectors as $\boldsymbol{a}_k$, $k = 1, 2, \ldots, n$. For the indexed vector set $\{\boldsymbol{a}_k\}$ of m-vectors $\{\boldsymbol{a}_k\} \in \mathbf{R}^{\mathrm{m}}$ and for any unit vector $\boldsymbol{q} \in \mathbf{R}^{\mathrm{m}}$ the energy E_q measured in direction $\boldsymbol{q}$, is defined as

$$E_{\boldsymbol{q}}[\boldsymbol{A}] = \sum_{k=1}^{n} \left(\boldsymbol{q}^T \cdot \boldsymbol{a}_k\right)^2. \tag{2}$$

More generally, the energy E_Q measured in a subspace $\mathrm{Q} \subset \mathbf{R}^{\mathrm{m}}$, is defined as

$$E_Q[\boldsymbol{A}] = \sum_{k=1}^{n} \left\|P_Q(\boldsymbol{a}_k)\right\|^2, \tag{3}$$

where $P_Q(\boldsymbol{a}_k)$ denotes the orthogonal projection of $\{\boldsymbol{a}_k\}$ into the subspace Q and $\|.\|$ denotes the euclidean norm. In other words, the oriented energy of a vector sequence $\{\boldsymbol{a}_k\}$, measured in the direction $\boldsymbol{q}$ (subspace Q) is the energy of the signal, projected orthogonally on to the vector $\boldsymbol{q}$ (subspace Q).

2.1.2 The Oriented Energy Concept and the SVD

In Section 2.1.1, attention was given to the basic concepts of the oriented energy distribution. In this section, the tools which allow numerical characterization of the oriented energy concept will be studied.

The singular value decomposition (SVD). The SVD for real matrices is based upon the following theorem [2], [3], [4]:

Theorem 1. *For any real $m \times n$ matrix $\boldsymbol{A}$, there exists a real factorization*

$$\boldsymbol{A} = \underset{\mathrm{m\times m}}{\boldsymbol{U}} \cdot \underset{\mathrm{m\times n}}{\boldsymbol{S}} \cdot \underset{\mathrm{n\times n}}{\boldsymbol{V}^T} \tag{4}$$

in which the matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ are real orthonormal, and matrix $\boldsymbol{S}$ is real pseudodiagonal with nonnegative diagonal elements.

The diagonal entries σ_i of $\boldsymbol{S}$ are called the singular values of the matrix $\boldsymbol{A}$. It is assumed that they are sorted in nonincreasing order of magnitude. The set of singular values $\{\sigma_i\}$ is called the singular spectrum of matrix $\boldsymbol{A}$. The columns $\boldsymbol{u_i}$ and $\boldsymbol{v_i}$ of $\boldsymbol{U}$ and $\boldsymbol{V}$ are called, respectively, the left and right singular vectors of matrix $\boldsymbol{A}$. The space $\mathrm{S}_{\mathrm{U}}^{\mathrm{r}} = \mathrm{span}\,[\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_r]$ is called the rth left principal subspace. In a similar way, the rth right singular subspace is defined.

Sensor	Description
0	Thumb flexure (lower joint)
1	Thumb flexure (second joint)
2	Thumb-index finger abduction
3	Index finger flexure (at knuckle)
4	Index finger flexure (second joint)
5	Index-middle finger abduction
6	Middle finger flexure (at knuckle)
7	Middle-ring finger (second joint)
8	Middle-ring finger abduction
9	Ring finger flexure (at knuckle)
10	Ring finger flexure (second joint)
11	Ring-little finger abduction
12	Little finger flexure (at knuckle)
13	Little finger flexure (second joint)

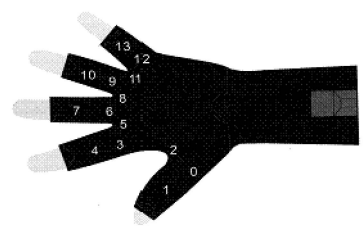

Fig. 2. Sensor Mappings for 5DT Data Glove 14 Ultra.

Proofs of the above classical existence and uniqueness theorems are found in [2].

Lemma 1. *The number of non zero singular values, equals the algebraic rank of the matrix $\boldsymbol{A}$.*

Lemma 2. *Via the SVD, any matrix $\boldsymbol{A}$ can be written as the sum of $r = \mathrm{rank}(\boldsymbol{A})$ rank-one matrices*

$$\boldsymbol{A} = \sum_{i=1}^{r} \boldsymbol{u}_i \cdot \boldsymbol{\sigma}_i \cdot \boldsymbol{v}_i^T, \tag{5}$$

where $(\boldsymbol{u}_i, \sigma_{i,}, \boldsymbol{v}_i)$ is the ith singular triplet of matrix $\boldsymbol{A}$.

Lemma 3. *Frobenius norm of $m \times n$ matrix $\boldsymbol{A}$ of rank r*

$$\|\boldsymbol{A}\|_F^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{a}_{ij}^2 = \sum_{k=1}^{r} \sigma_k^2, \tag{6}$$

where σ_k are the singular values of $\boldsymbol{A}$.

The total energy in a vector sequence $\{\boldsymbol{a}_k\}$ associated with matrix $\boldsymbol{A}$ as defined in Definition 1, is equal to the energy in the singular spectrum.

The smallest nonzero singular value corresponds to the distance in Frobenius norm, of the matrix to the closest matrix of lower rank. This property makes SVD attractive for approximation and data reduction purposes.

Conceptual relations between SVD and oriented energy. We are now in the position to establish the link between the singular value decomposition and the concept of oriented energy distribution.

Define the unit ball UB in $\mathbf{R}^{\mathrm{m}}$ as $\mathrm{UB} = \left\{\mathbf{q} \in \mathbf{R}^{\mathrm{M}} \middle| \|\mathbf{q}\|_2 = 1\right\}$.

Theorem 2. *Consider a sequence of m-vectors $\{\boldsymbol{a}_k\}$, $k = 1, 2, \ldots, n$ and the associated $m \times n$ matrix $\boldsymbol{A}$ with SVD as defined in (4) with $n \geq m$. Then,*

$$E_{\boldsymbol{u}_i}[\boldsymbol{A}] = \sigma_i^2 \tag{7}$$

$\forall \boldsymbol{q} \in \mathrm{UB}$: *if* $\mathbf{q} = \sum_{i=1}^{m} \gamma_i \cdot \mathrm{u}_i$, *then*

$$E_{\boldsymbol{q}}[\boldsymbol{A}] = \sum_{i=1}^{m} \gamma_i^2 \cdot \sigma_i^2. \tag{8}$$

Proof. Trivial from Theorem 1. □

The oriented energy measured in the direction of the ith left singular vector of the matrix $\boldsymbol{A}$, is equal to the ith singular value squared. The energy in an arbitrary direction $\boldsymbol{q}$ is the linear combination of "orthogonal" oriented energies associated with the left singular vectors. If the matrix $\boldsymbol{A}$ is rank deficient, then there exist directions in $\mathbf{R}^{\mathrm{m}}$ that contain no energy at all.

With the aid of Theorem 2, one can easily obtain, using the SVD, the directions and spaces of extremal energy, as follows:

Corollary 1. *Under the assumptions of Theorem 2:*

1. $\max_{\boldsymbol{q} \in UB} E_{\boldsymbol{q}}[\boldsymbol{A}] = E_{\boldsymbol{u}_1}[\boldsymbol{A}] = \sigma_1^2,$ (9)
2. $\min_{\boldsymbol{q} \in UB} E_{\boldsymbol{q}}[\boldsymbol{A}] = E_{\boldsymbol{u}_m}[\boldsymbol{A}] = \sigma_m^2,$ (10)
3. $\max_{Q^r \subset R^m} E_{Q^r}[\boldsymbol{A}] = E_{S_U^r}[\boldsymbol{A}] = \sum_{i=1}^{r} \sigma_i^2,$ (11)
4. $\min_{Q^r \subset R^m} E_{Q^r}[\boldsymbol{A}] = E_{\left(S_U^{m-r}\right)^{\perp}}[\boldsymbol{A}] = \sum_{i=m-r+1}^{m} \sigma_i^2,$ (12)

where "max" and "min" denote operators, maximizing or minimizing overall r-dimensional subspaces Q^r of the space $\mathbf{R}^{\mathrm{m}}$. S_U^r is the r-dimensional principal subspace of matrix $\boldsymbol{A}$ while $\left(\mathrm{S}_U^{m-r}\right)^{\perp}$ denotes the r-dimensional orthogonal complement of S_U^{m-r}.

Proof. Property (9), (10), (11), and (12) follow immediately from the SVD Theorem 1 and from Theorem 2. □

In other words, (9) and (10) relate the SVD to the minima and maxima of the oriented energy distribution. In fact, it can be shown that extrema occur at each left singular direction.

The rth principal subspace S_U^r is, among all r-dimensional subspaces of $\mathbf{R}^{\mathrm{m}}$, the one that senses a maximal oriented energy (11). Properties (11) and (12) show that the orthogonal decomposition of the energy via the singular value decomposition is canonical in the sense that it allows subspaces of dimension r to be found where the sequence has minimal and maximal energy. This decomposition of the ambient space, as direct sum of a space of maximal and minimal energy for a given vector sequence, leads to very interesting rank consideration.

By establishing this link between the oriented energy and SVD, we proved that the first r left singular vectors sensing the maximal energy of glove data matrix $\boldsymbol{A}$ and, thus, account for most of the variation in the original data. This means that with an $m \times n$ data matrix that is usually overdetermined with much more samples (columns) than channels (rows), $n \gg m$, the singular value decomposition allows most of signature characteristics to be compressed into r vectors.

2.2 Reference Signature

During the enrollment stage, 10 sample signatures from each writer to be enrolled are collected and pairwise angles between their principal subspaces are computed. Based on these angles, a reference signature is selected as the one that presents minimal overall angle to the others. The simulation results in Section 3 will show that the value of at least ten samples per reference signature result in the best performance for the proposed technique in term of its receiver operating characteristic (ROC).

2.3 Matching

Having modeled the signature through its rth principal subspace S_U^r in Section 2.1, the authenticity of the tried signature can be determined by calculating the angle between its principal subspace and the reference or template signature. This angle is referred to as similarity factor (SF) and given in percent. Different algorithms can be used in finding the angles between principal subspaces, The SVD-based algorithm for cosine is considered as the standard one at present and is implemented in software packages, e.g., in MATLAB, code *subspace.m*.

A complete description of the singular value decomposition (SVD)-based algorithm for computing cosines of principal angles between two subspaces can be found in [5], [6].

TABLE 1
The Average Amount of Truncated Energy as a Function of r

Dimension of principal subspace	Truncated energy
10	013%
9	0.19%
8	0.25%
7	0.33%
6	0.45%
5	0.57%
4	0.81%
3	1.01%
2	1.70%

3 Experimental Results and Discussion

3.1 Selecting the System Parameters

To verify the efficiency of the proposed technique in handwritten signature verification, the 5DT Data Glove 14 Ultra is used. This glove uses 14 sensors to measure finger flexure (two sensors per finger) as well as the abduction between each finger. The system interfaces with computer via cable to USB port or via Bluetooth technology (up to 20 m range). The SVD-signature verification algorithm is written in MATLAB 7.0 and run on a machine powered by Intel Core 2 Duo processor. The CPU time is about 80 ms.

In the first experiment, the best value for the dimension of principal subspace (r) as a trade-off between truncated energy of signature and reduced dimensionality of the data glove output matrix $\boldsymbol{A}$ is found. 100 data sets each contains 20 genuine signatures are collected. The 14 singular values of the 20×100 signatures are calculated and the average amount of truncated energy as a function of r is obtained and shown in Table 1.

Table 1 shows that the dimensionality of the data glove output matrix $\boldsymbol{A}$ can be significantly reduced without affecting the amount of energy oriented towards the principal subspace. Different values for r may be used in constructing the principal subspace, but as a trade off between reduced dimensionality and truncated energy, we use $r = 5$ for all authors. Using this value of r, we have 99.43 percent of the signature energy compressed into the five left singular vector of the data matrix $\boldsymbol{A}$. This will significantly reduce the required storage space for enrollment as well as the computational time for matching. At the same time, this will help reducing the differences between different attempts of the same signature due to the emotional state of the signing person.

Having modeled the signature through its five left-singular vectors of its data matrix $\boldsymbol{A}$, in the second experiment the decision threshold is sought. To achieve this goal, 100 data sets, each containing 110 genuine signatures from one signature contributor are obtained. The first 10 genuine signatures are used to find the reference signature (enrollment model) of the users and the remaining 100 genuine signatures are matched with it using the SVD-based signature verification technique. The similarity factors over the 100 data sets are calculated (100×100) and their relative frequency distribution over the class intervals is shown in Table 2.

It is clear form Table 2 that the threshold for authentic signature can be set at similarity factor of no more than 73 percent for zero false rejection rate (FRR) or at any value within the range from 73 percent to 80 percent for $\mathrm{FRR} = 2.5$ percent. This value is called the decision

TABLE 2
Relative Frequency Distribution of Similarity Factors of Genuine Signatures

Class interval	Relative frequency
less than 73%	0.000
73%-80%	0.025
80%-85%	0.075
95%-90%	0.210
90%-95%	0.330
higher than 95%	0.360

TABLE 3
Relative Frequency Distribution of Similarity Factors of Forgery Signatures

Class interval	Relative frequency
less than 25%	0.000
25%-45%	0.025
45%-50%	0.031
50%-55%	0.250
55%-60%	0.443
60%-65%	0.160
65%-70%	0.070
70%-75%	0.021
higher than 75%	0.000

threshold for authenticity since signatures with equal or higher values would be recognized by the system as genuine.

In the third experiment, 100 data sets, each containing 100 skilled forgeries to an authentic signature are obtained. The forgery signatures in each data set are compared with the reference signature of the writer and 100 similarity factors are obtained. The percentage distribution of the 100×100 similarity factors as a function of the class limits are shown in Table 3.

It is clear from the results in Table 3 that if the decision threshold for forgery is set at any value within the range from 70 percent to 75 percent, the proposed technique would recognize forgery signatures with false acceptance rate $(\mathrm{FAR}) \leq 2.1$ percent.

Combining the results obtained in these experiments it becomes quite reasonable to set the decision threshold of the SVD-signature verification technique at 75 percent. Threshold value of 75 percent provides the optimal trade-off between FAR and FRR such that both FAR and FRR will be low enough to fulfill the requirements of most of online signature verification applications.

3.2 Results

In the fourth experiment, the overall performance of the system is tested against the number of genuine signatures used in defining the reference signature of the writer. The FAR and FRR are calculated as a function of the number of genuine signatures and depicted against each other in terms of a receiver operating characteristic (ROC) curve. The FRR is calculated using the same data sets that are used in the second experiment. The FAR is calculated using the same data sets used in the third experiment. Fig. 3 shows the ROC curves of the above results for three different values for the number of the genuine signatures per reference signature.

As indicated in Fig. 3, the SVD-based signature verification technique clearly shows significant improvement in its performance as the number of signatures per reference signature increases. However, the best performance in term of ROC graph is obtained for the value of 10. For greater values than 10, there is no tangible improvement in the performance of the system. Fig. 3 also shows that with 10 genuine signatures per reference signature the Equal

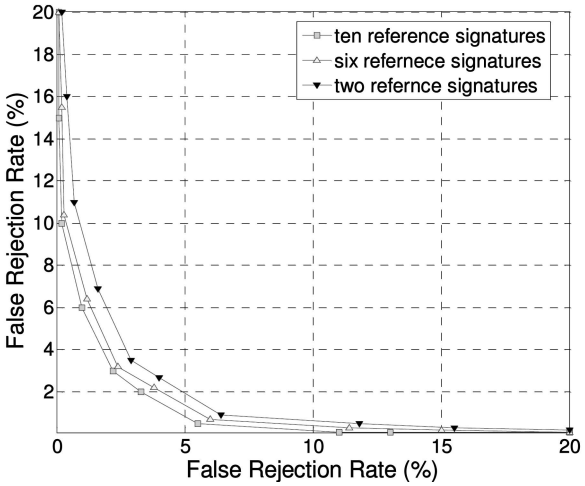

Fig. 3. ROC curves of the SVD-based signature verification technique as a function of number of samples per reference signature.

TABLE 4
EER for Different online Signature Verification Techniques

Technique	EER
Data glove (proposed)	2.37%
Muramatsu *et al* [7]	2.6%
Kholmatov *et al* [8]	2.8%
Nakanishi *et al* [9]	3.3%
Shinatro *et al* [10]	4.1%
Nakanishi et. Al [11]	4.2%
Fierrez-Aguilar *et al* [12]	from 5% to 7%
Fierrez-Aguilar *et al* [13]	7.14%
H. Lei et al [14]	7.2%

Error Rate (EER) of the proposed technique is about 2.37 percent when the decision threshold is set at 75 percent.

Since no other technique available for data glove online signature verification, listed in the Table 4 the performances of the most recently proposed techniques for pen tablets in terms of their EER achieved values. It is unfair to compare technique based on different input data devices, however the purpose of this comparison is to show the effectiveness of the proposed system as an emerging solution to the online signature verification problem.

In addition to the aforementioned verification techniques, the First International Signature Verification Competition (SVC 2004) [15] has tested 13 systems from industry and academia and found that the best equal error rate for class of skilled forgeries is 2.8 percent [8].

From the results shown in Table 4, it is clear that the proposed technique yields slightly lower EER value than the best tablet-based signature verification technique. However, we are sure that the achieved EER value can be further reduced if a data glove especially designed for signature verification is used.

3.3 Discussion

Since the output signals from the data glove are related to the bending angles of the five fingers and the roll and pitch of the hand and independent from the position of the pen or orientation of the signature on the paper or tablet, there is no need for position or orientation normalization with respect to the glove. This is contrast to other input devices used in all online signature verification techniques where normalization of the position and orientation of the signature is necessary and results in varying degrees of success in correcting for these situations. While it is true that the positioning

of sensors on the five fingers will affect the data matrix and make verification a more challenging task, in our experience over thousands of trials we noticed that the sensor positioning problem is not severe enough to defeat the position normalization. This became clear in the second experiment where about 97.5 percent of genuine signatures managed to produce similarity factors above 75 percent when matched with their reference signatures.

The variation in time span of signature from one trial to another is still a valid issue with the data glove. However, with this SVD-based signature verification technique, the size of the data glove output matrix $\boldsymbol{A}$ is reduced to a single dimension $(14 \times \mathrm{r})$ because of the data dimensionality reduction, hence there is no need for time normalization due to data reduction.

4 Conclusion

A novel approach to the signature verification problem using a data glove as input device to the online signature verification system is presented. The technique is based on the singular value decomposition in finding a set of singular vectors sensing the maximum energy of the signature. This limited set of vectors is referred to as the principal subspace of data glove output matrix $\boldsymbol{A}$ and used to model the signature. The angle between the different principal subspaces is used for signature classification. In the experimental section of the paper, the optimal value of r as a compromise between the reduced dimensionality and truncated energy of matrix $\boldsymbol{A}$ was found. Next, the decision threshold is sought and the value of 75 percent for the similarity factor was found to have sufficiently low FAR and FRR values for most of online signature verification applications. Later, using the ROC curves of the proposed technique we arrived at the value of 10 as the best value for the number of genuine samples per reference signature.

This research paper is an initial attempt in demonstrating the data glove as an effective high bandwidth data entry device for signature verification. We know that the data glove is expensive and not user-friendly, but the system has the potential to offer a high level of security for special applications, including banking and electronic commerce. Off course, the offered features by the present data glove can be reduced without affecting its performance in signature verification. This will result in lower cost of the system and make it available for the average consumer.

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