

## SVD-BASED SIGNATURE VERIFICATION TECHNIQUE USING DATA GLOVE

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Data glove is a new dimension in the field of virtual reality environments, initially designed to satisfy the stringent requirements of modern motion capture and animation professionals. In this paper, we try to shift the implementation of data glove from motion animation towards signature verification problem, making use of the offered multiple degrees of freedom for each finger and for the hand as well. The proposed technique is based on the Singular Value Decomposition (SVD) in finding  $r$  singular vectors sensing the maximal energy of glove data matrix  $\mathbf{A}$ , called principal subspace, and thus account for most of the variation in the original data, so the effective dimensionality of the data can be reduced. Having identified data glove signature through its  $r$ th principal subspace, the authenticity can then be obtained by calculating the angles between the different subspaces. The SVD-signature verification technique is tested with large number of authentic and forged signatures, showing remarkable level of accuracy in finding the similarities between genuine samples as well as those differentiated between genuine-forgery trials.

*Keywords:* Data glove; online signature verification; singular value decomposition.

### 1. Introduction

Biometry offers the potential for automatic personal verification, and differently from other means personal verification, biometric means are not based on the possession of something or the knowledge of information. Of the various biometrics, signature-based verification has the advantage that signature analysis requires no invasive measurement and is widely accepted since signature has long been established as the most diffuse means for personal verification in our daily life, including

applications in commerce, banking transactions, automatic fund transfers, etc. A wide variety of feature extraction and classification methods have been applied to the signature recognition. Two categories of verification systems are usually distinguished: offline and online systems for handwritten signature authentication and verification.

### **1.1. *Offline approaches for signature recognition***

In offline systems, the signature is captured once the writing processing is over, and thus only a static image is available. As for offline signature verification processing, most of the earlier work involves the extraction of features from the signature image by various schemes. Qi *et al.* used local grid features and global geometric features to build multiscale verification functions.<sup>12</sup> Sabourin *et al.* used an extended shadow code as a feature vector to incorporate both local and global information into the verification decision.<sup>13</sup> Fang *et al.* used positional variances of the one-dimensional projection profiles of signature patterns and the relative stroke positions of two-dimensional patterns.<sup>3</sup> Meenakshi *et al.* used a quasi-multiresolution technique using GCS (Gradient, Structural and Concavity) features for feature extraction.<sup>10</sup> Wang *et al.* used moment invariant method for offline signature recognition and verification system.<sup>18</sup>

### **1.2. *Online approaches to signature recognition***

Input devices in this category are either digitizing tables or smart pens and hand gloves. In *digitizing table-based systems* both global and local features that summarize aspects of signature shape and dynamics of signature production are used for signature verification. In *pen-based systems* a smart pen is used to collect data such as pen-tip positions, speeds, accelerations or forces while a person is signing. The invisible pen-up parts of the signature are used to construct a signature verification system. Trajectories left in pen-up situation, called “virtual strokes”, are used to extract the optimal features, which represent the personal characteristics of the authentic signature and affect the error rate greatly.<sup>4,9,11,14</sup>

Data glove is a new dimension in the field of virtual reality environments, initially designed to satisfy the stringent requirements of modern motion capture and animation professionals. It offers comfort, ease of use, a small form factor and multiple application drivers. The high data quality, low cross-correlation and high data rate make it ideal for realistic real-time animation.

In this paper, we try to shift the implementation of data glove from motion animation towards signature verification problem, making use of the offered multiple degrees of freedom for each finger and for the hand as well. This permits a user to communicate to the computer a far richer picture of his or her signature than with most other input devices.

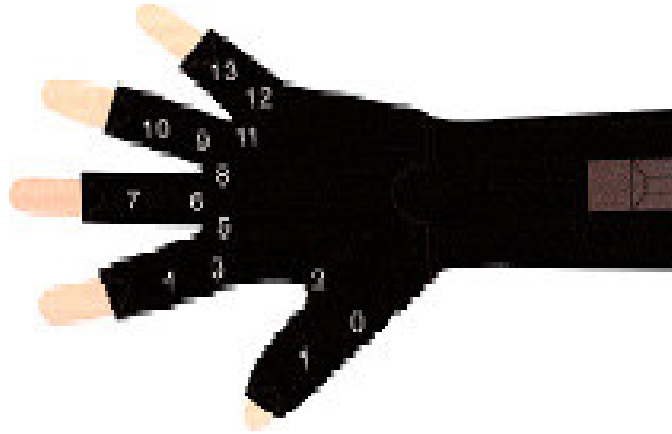


Fig. 1. Sensor mappings for 5DT data glove 14 ultra.

The dynamic features of the data glove provide information on:

1. Patterns distinctive to an individuals' signature and hand size.
2. Time elapsed during the signing process.
3. Hand trajectory dependent rolling.

Thus, the glove as a tool for signature recognition allows authentication of people not only through the biometric characteristics of their signatures but also through the size of their hands. Figure 1 shows the data glove with the location of the sensors.

In this paper, we present a novel technique based on the data glove for signature verification. The technique focuses on the features of signature in the principal components of the data matrix  $\mathbf{A}$ , so that the effective dimensionality of the data can be reduced. Next, the angle between the different principal subspaces is calculated and used as an indicator to the authenticity of the tried signature.

To make the presentation as clear as possible, an attempt is made to adhere to a somewhat standard notational convention. Lower case **boldface** characters will generally refer to vectors. Upper case **BOLDFACE** characters will generally refer to matrices.  $(.)^T$  will be used to denote the transpose operation.  $R^m$  denotes  $m$ -dimensional vector space of real  $n$ -tuples.

## 2. SVD-Based Signature Verification Technique

Consider a data glove of  $m$  sensors, each generates  $n$  samples per signature, producing an output data matrix,  $\mathbf{A}(m \times n)$ . Usually  $n \gg m$ , where  $m$  denotes the number of measured channels while  $n$  denotes the number of measurements. It has been found in many signal processing applications and control systems that the singular value decomposition of matrix formed from observed data can be used to improve methods of signal parameter estimation and system identification. In this section

we extend the implementation of SVD and the principal components of data matrix  $\mathbf{A}$  towards signature verification systems.

Principal component analysis, originating in some work by Karl Pearson around the turn of the last century and further developed in 1930s by Harold Hotelling, consist of finding an orthogonal transformations of the original — stochastic-variables to a new set of uncorrelated variables, which are derived in nonincreasing order of importance. These so-called principal components are the linear combination of the original variables, its first few components will account for most of the variation in the original data so the effective dimensionality of the data can be reduced.<sup>5</sup>

### 2.1. *The oriented energy concept and the SVD*

**Definition 1 (Oriented energy).** Let  $\mathbf{A}$  be a  $m \times n$  matrix and denote its  $n$  columnvectors as  $\mathbf{a}_k, k = 1, 2, \dots, n$ . For the indexed vector set  $\{\mathbf{a}_k\}$  of  $m$ -vectors  $\mathbf{a}_k \in R^m$  and for any unit vector  $\mathbf{q} \in R^m$  the energy  $E_q$  measured in direction  $\mathbf{q}$ , is defined as:

$$E_q[\mathbf{A}] = \sum_{k=1}^n (\mathbf{q}^T \cdot \mathbf{a}_k)^2. \quad (1)$$

More generally, the energy  $E_Q$  measured in a subspace  $Q \subset R^m$ , is defined as:

$$E_Q[\mathbf{A}] = \sum_{k=1}^n \|P_Q(\mathbf{a}_k)\|^2 \quad (2)$$

where  $P_Q(\mathbf{a}_k)$  denotes the orthogonal projection of  $\mathbf{a}_k$  into the subspace  $Q$  and  $\|\cdot\|$  denotes the Euclidean norm.

In other words, the oriented energy of a vector sequence  $\{\mathbf{a}_k\}$ , measured in the direction  $\mathbf{q}$  (subspace  $Q$ ) is nothing else than the energy of the signal, orthogonally projected on the vector  $q$  (subspace  $Q$ ).

**Theorem 1 [The singular value decomposition (SVD)].** *For any real  $m \times n$  matrix  $A$ , there exist a real factorization:*

$$\mathbf{A} = \underset{m \times m}{\mathbf{U}} \cdot \underset{m \times n}{\mathbf{S}} \cdot \underset{n \times n}{\mathbf{V}}^T \quad (3)$$

*in which the matrices  $\mathbf{U}$  and  $\mathbf{V}$  are real orthonormal, and matrix  $\mathbf{S}$  is real pseudo-diagonal with non-negative diagonal elements.<sup>9</sup>*

The diagonal entries  $\sigma_i$  of  $\mathbf{S}$  are called the singular values of the matrix  $\mathbf{A}$ . It is assumed that they are sorted in nonincreasing order of magnitude. The set of singular values  $\{\sigma_i\}$  is called the singular spectrum of matrix  $\mathbf{A}$ . The columns  $\mathbf{u}_i$  and  $\mathbf{v}_i$  of  $\mathbf{U}$  and  $\mathbf{V}$  are called respectively the left- and right-singular vectors of matrix  $\mathbf{A}$ . The space  $\mathbf{S}_U^r = \text{span}[\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_r]$  is called the  $r$ th left principal subspace. In a similar way, the  $r$ th right singular subspace is defined. Proofs of the above classical existence and uniqueness theorems are found in Ref. 14.

The oriented energy measured in the direction of the  $i$ th left singular vector of the matrix  $\mathbf{A}$ , is equal to the  $i$ th singular value squared.<sup>2,17</sup> The  $r$ th principal subspace  $S_U^r$  is, among all  $r$ -dimensional subspaces of  $R^m$ , the one that senses a maximal oriented energy. Thus, the orthogonal decomposition of the energy via the singular value decomposition is canonical in the sense that it allows to find subspaces of dimension  $r$  where the sequence has minimal and maximal energy. This decomposition of the ambient space, as direct sum of a space of maximal and minimal energy for a given vector sequence, leads to a very interesting rank consideration.

By establishing the link between the oriented energy and SVD, we proved that the first  $r$  left singular vectors sense the maximal energy of glove data matrix  $\mathbf{A}$ , and thus account for most of the variation in the original data. This means that with  $m \times n$  data matrix that is usually largely overdetermined with much more samples (columns) than channels (rows):  $n \gg m$  the singular value decomposition allows to compact most signature characteristics into  $r$  vectors.

Now, having identified each signature through its  $r$ th principal subspace  $S_U^r$ , the authenticity of the tried signature can be obtained by calculating the angle between its principal subspace and the authentic one.

## 2.2. Angle between principal subspaces (matching)

As in Sec. 2.1, the signature modeled through its  $r$ th principal subspace  $S_U^r$ , the authenticity of the tried signature can be obtained by calculating the angle between its principal subspace and the authentic one. This angle is refereed to as similarity factor (SF) and given in percent. Different algorithms can be used in finding the angles between principal subspaces. The SVD-based algorithm for cosine is considered as the standard one at present and is implemented in software packages, e.g. in MATLAB, version 5.3, 2000, code *subspace.m*.

A singular value decomposition (SVD)-based algorithm<sup>1,6,7</sup> for computing cosines of principal angles can be formulated as follows:

Let columns of matrices  $\mathbf{Q}_F \in R^{n \times p}$  and  $\mathbf{Q}_G \in R^{n \times q}$  form orthonormal bases for the subspaces  $F$  and  $G$ , respectively. The reduced SVD of  $\mathbf{Q}_F^T \mathbf{Q}_G$  is

$$\mathbf{Y}^T \mathbf{Q}_F^T \mathbf{Q}_G \mathbf{Z} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_q), \quad 1 \geq \sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_q \geq 0, \quad (4)$$

where  $\mathbf{Y} \in R^{p \times q}$ ,  $\mathbf{Z} \in R^{q \times q}$  both have orthonormal columns. Then the principal angles can be computed as

$$\theta_k = \arccos(\sigma_k) \quad k = 1, \dots, q, \quad (5)$$

where

$$0 \leq \theta_1 \leq \theta_2 \leq \dots \leq \theta_q \leq \frac{\pi}{2}. \quad (6)$$

## 2.3. Reference signatures

During the enrollment stage, ten sample signatures from each writer to be enrolled are collected and pairwise angles between their principal subspaces are computed.

Based on these angles, a reference signature is selected as the one that presents minimal overall angle to the others. The value of ten sample signatures is chosen in determining the reference signature, because it gives the best performance for the SVD-based signature verification technique in terms of receiver operating characteristic (ROC) curves, as Sec. 3 shows.

#### 2.4. Model for the SVD-based signature verification technique

The model for the proposed signature verification technique is shown in Fig. 2. The whole system is divided into two sections:

Enrollment section:

- Use data glove to provide the system with ten genuine samples of his/her signature.
- Out of the collected ten genuine samples select the reference signature.
- Extract the  $r$ -principal subspace of the reference signature and save it in the database for matching.

Verification section:

- Use data glove to input the signature of the user (one sample).
- Calculate the  $r$ -principal subspace of the claimed identity using SVD.
- Match the principal subspace of the claimed identity to the enrolled models in the database through the similarity factor.
- Compare the similarity factor with the decision threshold for ACCEPT or REJECT.

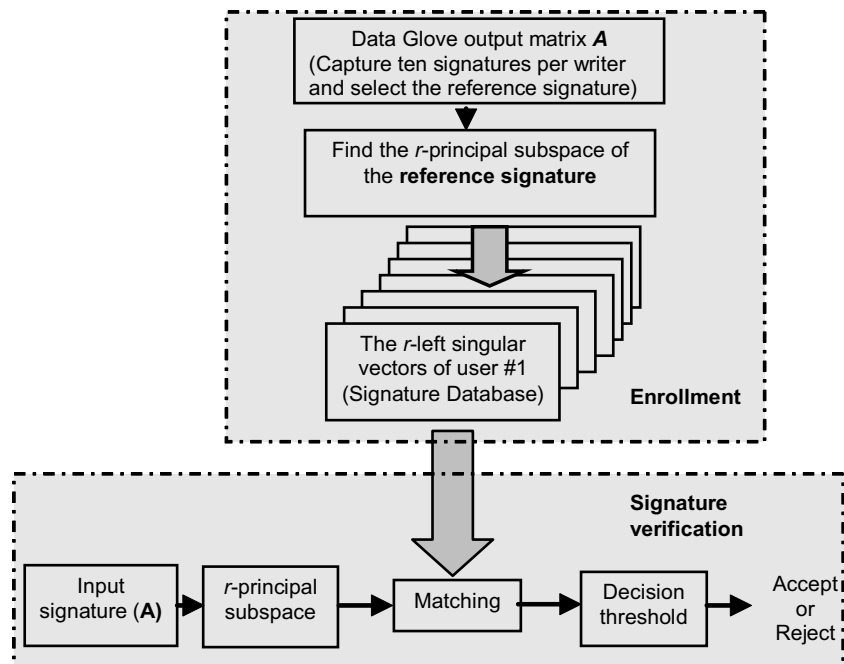


Fig. 2. Proposed model for the SVD-based signature verification technique.

### 3. Experimental Results

To verify the efficiency of the proposed technique in handwritten signature verification, the 5DT Data Glove 14 Ultra is used. This glove uses 14 sensors to measure finger flexure (two sensors per finger) as well as the abduction between fingers. Glove data is acquired using eight-bit resolution (256 positions) for each figure as well as for the axes of the tilt sensor roll and pitch. The roll and pitch angle of the glove are measured using tilt sensor through a  $\pm 60^\circ$  linear range. The 5DT Data Glove starts up in command mode. The full hand (five fingers, roll angle, pitch angle) can be sampled at least at 60 samples per second. The system interfaces with the computer via cable to USB port or via Bluetooth technology (up to 20 m distance). The SVD-signature verification algorithm is written in MATLAB 7.0 and run on a machine powered by Intel Core 2 Duo processor. The CPU time is about 80 ms.

Since, the output signals of data glove are related to the bending angles of the five fingers and the roll and pitch of hand, and accordingly independent from the position of pen or orientation of signature on the paper of tablet, there is no need for position or orientation normalization with data glove. This is contrary to other input devices, where normalization of signature in position and orientation is a basic block in all online signature verification techniques with different degrees of success in correcting these situations. The varying time span of signature is still a valid issue with data glove. However, since all data glove output matrices are reduced to the same dimensionality by the SVD-based signature verification technique, the elapsed time during the signing process is considered part of the dynamic features of signature and no time normalization is used.

In order to find the best value of  $r$  as a trade-off between truncated energy of signature and reduced dimensionality of the data glove output matrix  $\mathbf{A}$ , 100 data sets each containing 20 genuine signatures are collected. The 14 singular values of the  $20 \times 100$  signatures are calculated and the average amount of the truncated energy is obtained as a function of  $r$  and tabulated in Table 1.

It is quite clear from Table 1 that the dimensionality of the data glove output matrix  $\mathbf{A}$  can be significantly reduced without affecting the amount of energy

Table 1. The average amount of truncated energy as a function of  $r$ .

Dimension of the Principle Subspace ( $r$ )	Truncated Energy %
10	0.13
9	0.19
8	0.26
7	0.33
6	0.45
5	0.57
4	0.81
3	1.01
2	1.70

oriented towards the principal subspace. Different values for  $r$  can be used in constructing the principal subspace, but as a trade off between reduced dimensionality and truncated energy, we suggest the use of the value  $r = 5$  for all authors. With this value of the dimensionality of the principal subspace we only sacrifice 0.57% of signature variation for reducing the rank of  $\mathbf{A}$  from 14 to 5.

In the first experiment the threshold for authentic signatures is sought. 100 data sets, each containing 110 genuine signatures from one signature contributor, are obtained. The first ten genuine signatures are used to find the reference signature of each user. The remaining 100 genuine signatures are run with the SVD-based signature verification technique for verification. The percentage distribution of the  $100 \times 100$  similarity factors as a function of the class limits is depicted in Fig. 3.

It is quite clear from the results in Fig. 3 that the threshold of authenticity can be set at 75% or slightly lower. With this threshold value the proposed technique would be able to recognize genuine signatures with false rejection rate (FRR)  $\leq 2.5\%$ . The 75% value for the similarity factor is called the threshold of authenticity because signatures that produce higher values when compared with their reference signatures would be recognized as genuine.

In the second experiment, 100 data sets, each containing 100 skilled forgeries to an authentic signature are obtained. The forgery signatures in each data set are compared with the reference signature of the writer and 100 similarity factors are obtained. The percentage distribution of the  $100 \times 100$  similarity factors as a function of the class limits is depicted in Fig. 4.

It is quite clear from the results in Fig. 4 that if the threshold of forgery is set at 75%, the proposed technique will be able to recognize forgery signatures with false acceptance rate (FAR)  $\leq 1.2\%$ .

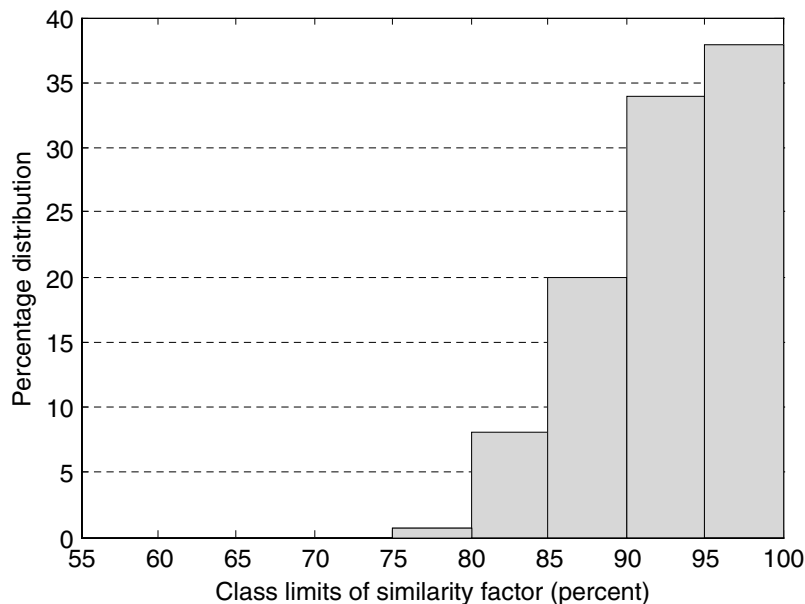


Fig. 3. Percentage distributions of class limits of similarity factors of genuine signatures.



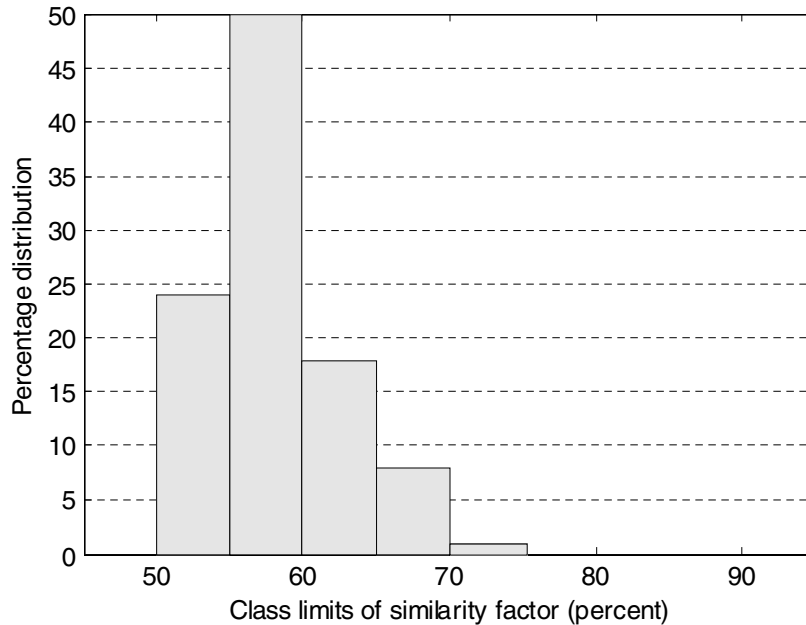


Fig. 4. Percentage distributions of class limits of similarity factors of forgery signatures.

From the results in Figs. 3 and 4, it becomes reasonable to set the decision threshold of the proposed technique at 75%. With this value of the decision threshold, we have both FAR and FRR of low enough values to fulfill the requirements of most online signature verification applications.

Now, having the threshold set at a similarity factor of 75%, the tried signatures which produce similarity factors greater than 75% are considered genuine, whereas signatures with less similarity factors are considered forgeries.

Some of genuine signatures and their skilled forgeries are shown in Fig. 5.

In the third experiment, the overall performance of the system is tested against the number of genuine signatures used in defining the reference signature of the writer. The FAR and FRR are calculated as a function of the number of genuine signatures and depicted against each other in terms of receiver operating characteristic (ROC) curve. The FRR is calculated using the same data sets that are used in the first experiments. The FAR is calculated using the same data sets used in the second experiment.

Figure 6 shows the ROC curves of the above results for three different values of the number of the genuine signatures that are used in determining the reference signature of each writer.

As Fig. 6 clearly indicates, the SVD-based signature verification technique shows significant improvement in its performance as the used number of genuine signatures in determining the reference signature is increased from two to ten. However, for values higher than ten, there is no tangible improvement in the performance of the system. Based on this result, we use the value 10 for the number of genuine samples per reference signature in the proposed technique.

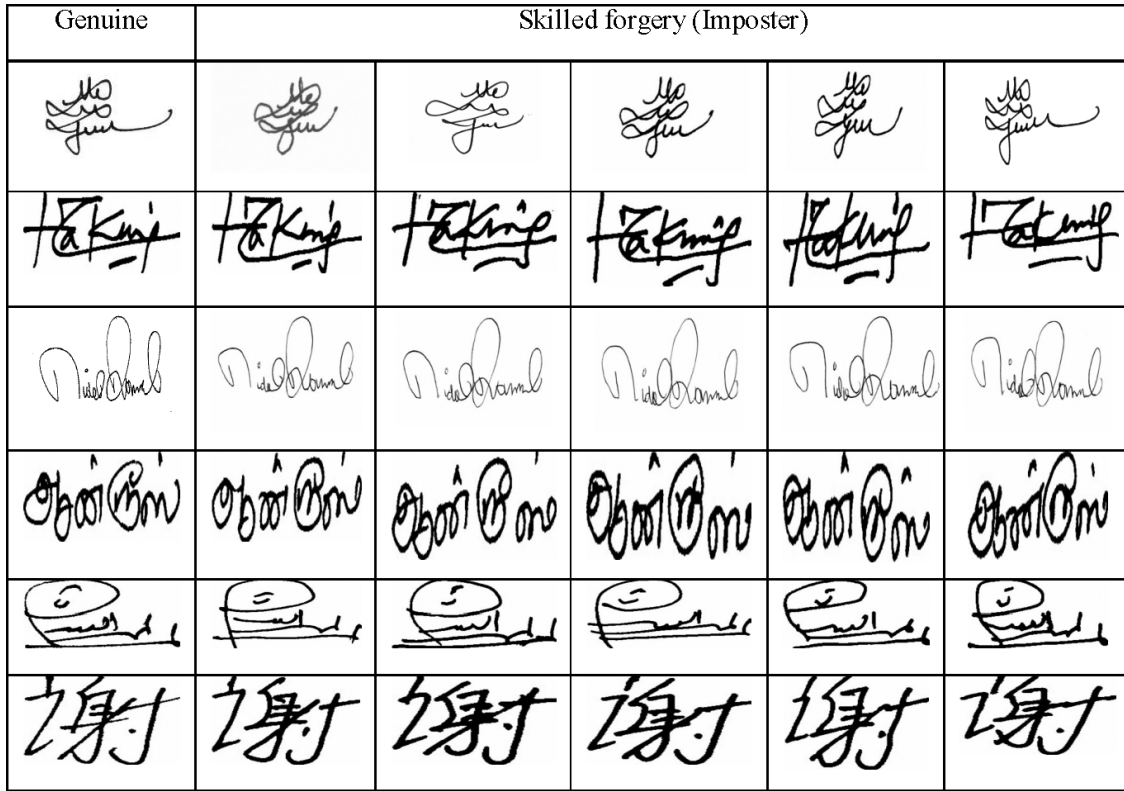


Fig. 5. Sample of genuine signatures and their skilled forgeries.

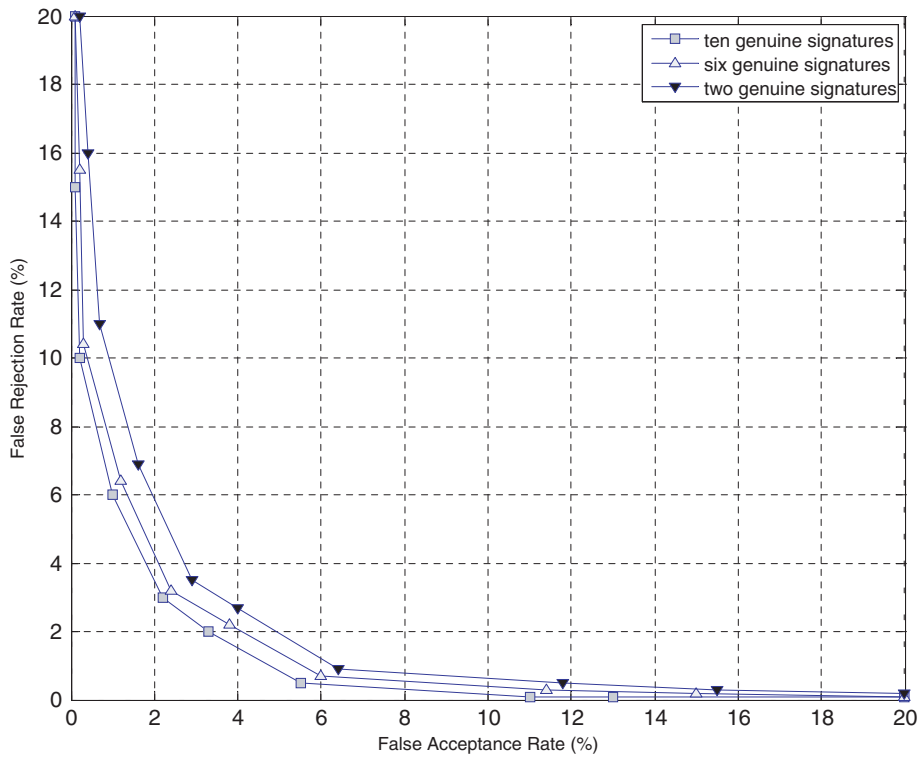


Fig. 6. ROC curves of the SVD-based signature verification technique as a function of the number of genuine samples that are used in determining the reference signature.

Figure 6 also shows that with ten genuine signatures per reference signature the Equal Error Rate (EER) of the proposed technique is about 2.37 when the threshold is about 0.75.

#### 4. Comparison with Other Online Methods

Since it is difficult to make a comparison between different signature verification techniques based on different data bases, we just list the achieved performance by some recently suggested signature verification techniques. An automatic handwritten signature verification system based on a serial multiexpert architecture obtained false acceptance rate (FAR) and false rejection rate (FRR) for skilled forgeries 19.8% and 2.04% respectively, and EER of 10.92%.<sup>15</sup> Online handwritten signature verification system using Hidden Markov Model (HMM) has achieved a false acceptance rate (FAR) of 4% and a false rejection rate (FRR) of 12% for both random and skilled forgeries and ERR of 11.5%.<sup>16</sup> In addition to the aforementioned verification techniques, the First International Signature Verification Competition (SVC2004),<sup>19</sup> has tested more than 15 systems from industry and academia and found that the best equal error rate is 2.8%, achieved in Ref. 8.

Comparing the proposed technique with others, the equal error rate value of the SVD-based signature verification technique (EER = 2.37%) is significantly lower than all the recently published techniques.

#### 5. Conclusion

A novel approach to signature verification problem has been presented. The technique is based on the singular value decomposition in finding  $r$ -singular vectors sensing the maximum energy of the tried signature, and thus account for most of variation. This will effectively reduce the dimensionality of the data. The angle between the  $r$ -principal subspaces of the different signatures is used as an indicator to the authenticity of the tried signature and referred to as similarity factor. Two experiments are conducted, in the first one twenty authentic trials of 40 writers are collected and the similarity factors among them are calculated. The SVD-based signature verification technique showed good capability in recognizing the authentic features among tried samples by developing similarity factors  $\gg 80\%$ . In the second experiment, 200 forgery samples per writer were collected and the similarity factors with authentic ones were calculated. The SVD-signature verification technique developed similarity factors  $\ll 70\%$  showing good structure in identifying the unauthentic features in the tried forgery samples. If we add to the efficient signature identification characteristics the low computational time, the SVD-signature verification technique represents a new trend in real-time signature verification systems.

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